Closing Wed: HW_9A, 9B, 9C
Final Exam, Saturday, March 11
Kane 130, 1:30-4:20pm

9.4 Diff. Eq. Apps (continued)

Mixing Problems:

Assume a vat of liquid has a substance entering at some rate and exiting as some rate, then

"The rate of change of the substance is equal to the rate at which the substance is coming IN minus the rate at which the substance is going OUT."

Here is what these problems typically look like:

V	= volume of the vat	(liters)
t	= time	(minutes)
$y(t)$	= amount in vat	(kg)
$\frac{dy}{dt}$	= rate	(kg/min)

Thus,

$$\frac{dy}{dt} = \text{Rate In} - \text{Rate out}$$

$$= \left(?\frac{kg}{L}\right)\left(?\frac{L}{min}\right) - \left(\frac{y}{V}\ \frac{kg}{L}\right)\left(?\frac{L}{min}\right)$$

$$y(0) = ?\ \text{kg}$$

Example:

Assume a 100 Liter vat contains 5kg of salt initially. Two pipes (A & B) pump in salt water (brine).

Pipe A: Enters at 3L/min with a concentration of 4kg/L of salt.

Pipe B: Enters at 5L/min with a concentration of 2kg/L of salt.

The vat is well mixed.

The mixture leaves the vat at 8L/min.

Let $y(t)$ = the amount of salt in the vat at time t.

(a) Find $y(t)$.
(b) Find the limit of $y(t)$ as $n \to \infty$.

The Logistics Equation

Consider a population scenario where there is a limit to the amount of growth (spread of a rumor, for example).

Let $P(t)$ = population size at time t.

M = maximum population size. (capacity)

We want a model that

a. …is like natural growth when P(t) is significantly smaller than M;
b. …levels off (with a slope approaching zero), then the population approaches M.

One such model is the so-called logistics equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) \text{ with } P(0) = P_0$$

Random "scary-looking" problems

Spring 2011 Final:

Brief summary of what it says:

$v(t)$ = velocity of an object

$$F = mg - kv$$

Recall:

$$F = ma = m\frac{dv}{dt}$$

You are given m, g, and k and asked for solve for $v(t)$.

Lecture:

Melting snowball, you were told that rate of change of volume is proportional to surface area. Thus,

$$\frac{dV}{dt} = -2k\pi r^2$$

and you were told to differentiate $V = \frac{4}{3}\pi r^3$ with respect to *t* and give a differential equation for *r*

Spring 2014:

A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day. You may assume that the pesticide mixes thoroughly with the water in the lake, and you should ignore other effects such as evaporation.

Winter 2011

Your friend wins the lottery, and gives you P_0 dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously (a pretty good approximation to reality if you make regular frequent withdrawals) at a rate of $3600 per year.

Fall 2009

The swine flu epidemic has been modeled by the Gompertz function, which is a solution of

$$\frac{dy}{dt} = 1.2\, y\, (K - ln(y)),$$

where $y(t)$ is the number of individuals (in thousands) in a large city that have been infected by time t, and K is a constant.

Time t is measured in months, with t = 0 on July 9, 2009.
On July 9, 2009, 75 thousand individuals had been infected.
One month later, 190 thousand individuals had been infected.